Fault detection based on higher-order sliding mode observer for a class of switched linear systems

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Abstract: This study considers the fault detection problem for a class of switched linear systems when the observer matching condition is not satisfied. Without using decoupling transformations, a residual signal, based on reduced order sliding mode observers, is introduced for the detection of faults which occur on the continuous part of the system. Two numerical simulations on the cascade multilevel converter and on the boost converter illustrate the effectiveness of the proposed fault detection scheme.

1 Introduction
Switched systems were widely studied in the literature because they are found in various fields such as in manufacturing, power, automotive and chemical process industries. A common definition is that they are modelled as a combination of both continuous-time dynamics and discrete event ones [1]. In the last two decades, many works have been made with regards to the stability and stabilisation of such systems [2, 3].

Associated with this class of systems, the observability and observer design problems have been investigated using different approaches. Assuming that each subsystem is observable, some works have supposed that the discrete state is known [4]. Others have considered both discrete and continuous state estimation using cascade observers [5] or stack of N observers (where N is the number of subsystems) [6]. Most of the above works did not address the observation problem for switched systems subject to unknown inputs.

Occurrence of faults can lead to critical situations for systems and can be extremely detrimental, not only to the equipment and surroundings, but also to the human operator if they are not detected in time. Associated with a physical system, a fault can be linked to a sensor deviation, an ageing of internal component, an actuator degradation etc. Considering the class of switched systems, a fault may occur on the discrete or and the continuous part of the system. Hence, it may influence the operating modes (spontaneous switching for instance) or change the dynamics of the continuous state.

Fault detection and isolation (FDI) have been widely investigated using various methods [7]. Observer-based FDI techniques rely on the estimation of outputs from measurements with the observer in order to detect the fault. They are usually based on either the analysis of appropriate residuals or an estimation of the fault variables. In [8], a \( H_{\infty} \) observer has been designed in order to ensure fault detection for a linear switched system with disturbances when the active mode is unknown. In [9], a stack of robust Luenberger observers and a discrete algorithm for the diagnosis have been combined to guarantee sensor FDI. Nevertheless, these schemes cannot be applied when the switched system has modes in which the state is not fully observable. Some physical systems, such as the multilevel converter or the boost converter, for instance, can be defined by non-observable subsystems and the existing approaches cannot be applied. Nevertheless, even though the individual modes are not observable in the classical sense, it is still possible to detect a fault using the measurements over an interval which involves multiple switching times.

An observer can be also synthesised to directly estimate a fault. Many techniques have been developed for linear systems with constant fault distribution matrices. Several FDI approaches based on sliding mode observer have been developed [10–13] for linear systems and a limited class of non-linear systems. A fault reconstruction method based on an adaptive sliding mode observer has been introduced for non-linear systems with parametric uncertainties [11].

Recently, some works have dealt with the fault estimation for switched systems. In [14, 15], based on a condition of strong detectability, an observation scheme for a class of switched linear systems with unknown inputs has been designed. In [16], a high-order sliding mode observer has been proposed to solve the problem of continuous and discrete state estimation for a class of observable non-linear switched systems with unknown inputs. In [17], the state observation and unknown input estimation of a class of switched linear systems with unknown inputs have been addressed. In [18], based on structural properties of the linear switched systems with faults, the state space has been decomposed in order to analyse the effect of the unknown inputs. Higher-order sliding-mode-based fault detection strategy and fault identification via Volterra integral equation have been designed. The schemes described in [16, 17] require that the observer matching condition is fulfilled. This condition is equivalent to the relative degree assumption. However, many physical systems do not satisfy this condition. This fact limits the practical applications of unknown input observers based on the observer matching condition.

The main contribution of this paper is to detect faults for switched linear systems, without the observer matching condition being satisfied, and when the subsystems are not assumed to be observable in the classical sense. Contrary to [17], no decoupling transformation is required to detect faults which occur on the continuous part of the system. Here, a higher-order sliding mode observer is designed to robustly estimate the observable components of the estimation error for each subsystem. Based on this estimation, a detector can alarm us from the appearance of the fault. Illustrative simulation results on a three cells converter and a boost converter show the effectiveness of the proposed scheme in order to detect faults corresponding to a capacitor ageing.

The outline of the paper is as follows. Section 2 deals with the problem formulation. In Section 3, the fault detection scheme is introduced based on an hybrid observer and an higher-order sliding
mode observer. Section 4 presents the effectiveness of the proposed strategy through simulation results on the multicellular converter and on the boost converter.

Through the paper, the following notations will be used. Let $A$ be a matrix $n \times n$. $A^T$ is the transpose of $A$. $\text{Ker}(A)$ is the null space of the transformation associated with $A$, that is $\text{Ker}(A) = A^{-1}\{0\}$. $\text{Im}(A)$ is the range of the transformation associated with $A$. $A^T$ is the left pseudo inverse of $A$.

2 Problem statement

Consider the following class of linear switched systems with faults occurring on the continuous part of the system

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + K_{\sigma(t)}f_c(t)$$
$$y(t) = C_{\sigma(t)}x(t) + D_{\sigma(t)}u(t)$$

(1)

where $x(t) \in \mathbb{R}^n$ is the continuous state, $y(t) \in \mathbb{R}^p$ is the output and $u(t) \in \mathbb{R}^k$ is the known input. The switching signal $\sigma : \mathbb{R}^+ \rightarrow \{1, \ldots, Q\}$ is assumed to be known and satisfies the minimal dwell time condition, denoted $T_\sigma$. It is a piecewise constant and right-continuous function that changes its values at switching times $t_k$, $k \in \mathbb{N}$. The so-called discrete state $s(t) \in \{1, \ldots, Q\}$ determines the actual dynamics in the possible $Q$ operating modes which corresponds to a specific instance of matrices $A_{\sigma_{t}}$, $B_{\sigma_{t}}$, $C_{\sigma_{t}}$, $D_{\sigma_{t}}$ and $K_{\sigma_{t}}$. That is to say

$$\sigma(t) = \sigma_k, \quad t_k-1 \leq t < t_k$$

(2)

where $t_0 = 0$ and $[t_k]$ are the switching time instants.

Term $f_c(t) \in \mathbb{R}^m$ represents the vector of fault which only influences the continuous dynamics of the switched system. It can be used to model system failures or actuator faults. Contrary to [17], the following conditions are not required

$$r_c < p \quad \text{Rank}(C_{\sigma_{t}}K_{\sigma_{t}}) = \text{Rank}(K_{\sigma_{t}}) = r_c$$

(3)

Therefore, one cannot easily find a non-singular state and output change of coordinates that decouples the fault $f_c(t)$ from a subset of the transformed coordinate.

The aim of this paper is to generate a residual vector which is sensitive to continuous faults $f_c$ despite condition (3) is not satisfied.

3 Fault detection based on sliding mode observer

Here, two observers are designed and combined to detect a fault on the continuous state for linear switched systems. The first one, motivated by Tanwani et al. [19], gathers partial information from individual modes of the switched system in order to reconstruct the continuous state. The second one is based on higher-order sliding mode theory to robustly estimate the observable components of the estimation error for each subsystem. Reduced order sliding mode observers are designed to be sensitive to a fault and act as residual signals for detection of faults. Before designing the observer, let us introduce further conditions.

Assumption 1: It is assumed that:

- the Euclidean norm of $u, f_c$ and their time derivatives are upper-bounded by known positive constants;
- system (1) is bounded input bounded state and satisfies the minimal dwell time definition;
- the switching signal $\sigma$ is known and persistent;
- system (1) is persistently observable according to Theorem 1. Therefore, there is $\eta \in \mathbb{N}$ such that

$$\mathcal{N}_{\eta}^k = \{0\}, \quad \forall k \geq \eta + 1$$

(6)

- the fault $f_c$ is detectable, that is its occurrence, independent of its size and type, would cause a change in the nominal behavior of the system. Furthermore, the occurrence of a continuous fault $f_c$ does not influence the observability of the switched system.

Considering Assumption 1, the following hybrid observer is proposed

$$\hat{x}(t) = A_{\sigma(t)}\hat{x}(t) + B_{\sigma(t)}u(t)$$
$$\dot{\hat{x}}(t_k) = \hat{x}(t_k^-) - \xi^k$$
$$\tilde{y}(t) = C_{\sigma(t)}\hat{x}(t) + D_{\sigma(t)}u(t)$$

(7)

The correction vector $\xi^k$ is computed using accumulated partial state information, obtained by the sliding mode observer, in order to ensure the convergence to zero of the error

$$e_x = \hat{x} - x$$

in the absence of fault.

The time derivative of the estimation error can be written as

$$\dot{e}_x(t) = A_{\sigma_{t}}e_x(t) - K_{\sigma_{t}}f_c(t), \quad t \in [t_{k-1}, t_k)$$
$$e_x(t_k) = e_x(t_k^-) - \xi^k$$
$$\tilde{y}(t) = C_{\sigma_{t}}e_x(t)$$

(8)

with $\hat{y} = \tilde{y} - y$.

Partial observers are designed to estimate the observable part of $e_x$ at time $t_k$ in spite of the presence of perturbation $f_c$. From the
observability matrices $H_{a_k}$ associated with each pair $(A_{a_k}, C_{a_k})$, let us choose matrices $Z_{a_k}$ (resp. $H_{a_k}$) such that its columns are an orthonormal basis of Im($H_{a_k}$) [resp. ker($H_{a_k}$)]. Hence, one gets \( \text{Im}(Z_{a_k}) = \text{Im}(H_{a_k}^T) \). From this construction, on the time interval \( [t_{k-1}, t_k) \), one can denote

\[
\begin{align*}
z(t) &= (Z_{a_k})^T e_{a_k}(t) \\
w(t) &= (W_{a_k})^T e_{a_k}(t) \\
S_k(Z_{a_k}^T) &= (Z_{a_k})^T A_{a_k} \\
R_k(Z_{a_k}^T) &= C_{a_k}
\end{align*}
\]

where \( z(t) \in \mathbb{R}^h \) represents the observable part of \( e_{a_k}(t) \), \( w(t) \in \mathbb{R}^{n-h} \) is its unobservable part, \( S_k \in \mathbb{R}^{h \times h} \) and \( R_k \in \mathbb{R}^{p \times h} \). Due to Assumption 1, the unobservable components, represented by \( w(t) \), are considered to be stable. From (8), one obtains

\[
\begin{align*}
\dot{z}(t) &= S_k z(t) - (Z_{a_k})^T K_{a_k} f_j(t) \\
\dot{y}(t) &= [\dot{y}_1(t), \ldots, \dot{y}_p(t)] = R_k z(t)
\end{align*}
\]

With this representation, the pair \( (S_k, R_k) \) is observable. The corresponding observability matrix \( H_k \) has a full rank.

It should be noted that the perturbation \( f_j \) acts on the dynamics of \( z \) in (10). Hence, the partial observer must provide an accurate estimate of \( z \) in spite of the presence of fault. Due to its robustness properties, a high-order sliding mode observer is proposed

\[
\begin{align*}
\dot{\hat{z}}(t) &= S_k \hat{z}(t) - \hat{R}_k^{-1} v(t), \quad \forall t \in (t_{k-1}, t_k) \\
\hat{z}(t_{k-1}) &= 0
\end{align*}
\]

where \( v(t) \) will be defined hereafter.

From Assumption 1, one can derive \( p \) known constants \( M_i > 0 \), \( \forall i = 1, \ldots, p \), such that Eq. (12) holds

The correction term of the observer (11) is defined by

\[
v = -\alpha_i M_i^{1/\nu} [R_{i_k} \hat{z} - \hat{y}_i]^T (R_{i_k} \hat{z} - \hat{y}_i) / \nu_i \delta_{i_k} + \alpha_i M_i^{1/\nu} [v_i - \hat{v}_i]^T (v_i - \hat{v}_i) / \nu_i \delta_{i_k} + \cdots + \alpha_i M_i^{1/\nu} [v_p - \hat{v}_p]^T (v_p - \hat{v}_p) / \nu_i \delta_{i_k} \]

The high-order sliding mode differentiator (see [16] for further details about the use of high-order sliding mode for multi-output systems) is used as an auxiliary dynamics.

**Theorem 2:** Using observer (11) with the correction term (13) and providing that constants $\alpha_i$ are chosen recursively and sufficiently large and constants $M_i$ satisfy condition (12), it is guaranteed that the state estimation error $e_{a_k}(t) = \hat{z}(t) - z(t)$ is as follows

\[
e_{a_k}(t) = 0, \quad \forall t \in [t_{k-1} + T^*, t_k)
\]

even if a fault has occurred, where $T^* > T^*_f$.

**Proof:** From (8) and (9) and using the transformation $\zeta = \hat{R}_k z$, the uncertain system (10) can be transformed into the following triangular observable form: \( \forall j = 1, \ldots, p \)

\[
\begin{align*}
\dot{\zeta}_j &= \hat{S}_j \zeta_j + \hat{T}_{j,k} (\zeta_j, f_j) \\
\hat{y}_j &= [1 \ 0 \ \ldots \ 0] \zeta_j
\end{align*}
\]

with \( \zeta = [(\zeta_1)^T \ldots (\zeta_p)^T]^T, \ \zeta_j \in \mathbb{R}^{l_j} \)

\[
\begin{bmatrix}
\hat{S}_{1,k} & 0 & 0 & \cdots & 0 \\
0 & \hat{S}_{2,k} & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & 0 & \hat{S}_{p,k}
\end{bmatrix} \in \mathbb{R}^{l_j \times l_j}.
\]

According to [20], the observer defined as follows

\[
\hat{z} = \hat{S}_k \zeta + v
\]

with $\hat{T}_{j,k}(z, f_j) \in \mathbb{R}^{l_j}$ defined in (15bis)

\[
\hat{S}_k = \begin{bmatrix}
\hat{S}_{1,k} & 0 & 0 & \cdots & 0 \\
0 & \hat{S}_{2,k} & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & 0 & \hat{S}_{p,k}
\end{bmatrix}
\]

yields the finite-time stabilisation of the observation error $\hat{z} - \zeta$. Therefore, using $\zeta = \hat{R}_k z$, one can conclude that observer (11), with (13), guarantees (14) even if a fault has occurred.

The approximation of the estimation error is transported to time \( t_k^- \) using the following stationary transition matrix

\[
\Psi(t_k^-, t_j^-) = e^{A_{\nu_i} t_k^-} e^{A_{\nu_i-1} t_{k-1^-}} \cdots e^{A_{\nu_i-1} t_{j-1^-}}, \quad k > j
\]

where $\Psi(t_k^-, t_k^-) = I$ and the switching period is $\tau_k = t_k - t_{k-1}$.

\[
\begin{align*}
R_{1,k}(S_k)^T (\hat{z} - z) + \sum_{i=0}^{h_k-1} R_{1,k}(S_k)^T (Z_{a_k})^T K_{a_k} f_j^{(h_k-1-i)} &< M_1 \\
\vdots & \\
R_{p,k}(S_k)^T (\hat{z} - z) + \sum_{i=0}^{h_k-1} R_{p,k}(S_k)^T (Z_{a_k})^T K_{a_k} f_j^{(h_k-1-i)} &< M_p
\end{align*}
\]
Following [19], let us introduce vector $\Omega^k$ defined in (18) where $\Theta^k$ is such that $\forall k \geq 1$, $\forall q = k - \eta, \ldots, k$

$$
\operatorname{Im}(\Theta^k) = \operatorname{Im} \left( \Psi(t_k^-, t_q^-) W_{\sigma(t_q^-)} \right)^\perp $$

if $q = k - \eta, \ldots, k - 1$

$$
\operatorname{Im}(\Theta^k) = \operatorname{Im} \left( W_{\sigma(t_k^-)} \right)^\perp
$$

Therefore, using the information on $[t_{k-\eta}, t_k)$, the correction vector can be defined as

$$
\xi^k = (\Theta^k)^T \Omega^k
$$

where

$$
\Theta^k = [\Theta^{k,k} \ldots \Theta^{k-\eta,k}]
$$

has rank $n$ from Assumption 1.

**Theorem 3:** Let us consider system (1) satisfying Assumption 1. The hybrid observer (7) provides an estimate of $x$ in finite time if no fault has occurred ($f_x(t) = 0$), that is

$$
\| \hat{x}(t) - x(t) \| = 0, \quad \forall t \geq t_\eta
$$

If a fault has occurred ($f_x(t) \neq 0$), the estimate of $x$ is such that

$$
\| \hat{x}(t) - x(t) \| > \epsilon, \quad \forall t \geq t_\eta
$$

where $\epsilon$ is a positive constant.

**Proof:** Let us first study the case without fault, that is $f_x(t) = 0$. Following the work in [19], one can estimate $e_x(t_k)$ as follows

$$
e_x(t_k) = - \left[ M^{k,k} \ldots M^{k-\eta,k} \right]
\left[
\begin{array}{c}
Z_{0\eta} e_x(t_k^-) \\
\vdots \\
Z_{0_{k-\eta}} e_x(t_k^-)
\end{array}
\right]
$$

with $M^{j,k} = (\Theta^j)^T \Psi(t_k^-, t_j^-), j = k - \eta, \ldots, k$.

According to Theorem 2, the state estimation error is as follows

$$
e_x(t) = 0, \quad \forall t \in [t_{k-1} + T^1_f, t_k)
$$

Therefore, from (24) and (25), one can obtain

$$|e_x(t_k)| = 0
$$

Using (26), one can conclude

$$\| \hat{x}(t) - x(t) \| = 0, \quad \forall t \geq t_\eta
$$

It concludes the finite-time stability of the state estimation error trajectory when no fault has occurred.

Let us now discuss the case when a fault has occurred. From (8), one can see that the time derivative of the estimation error is directly influenced by the fault. Since a fault is detectable, its occurrence causes a change in the nominal behaviour of the system. Furthermore, for the computation of $\xi^k$, the projection matrices $\Psi(t_k^-, t_k^-)$ do not take into account the fault and its evolution. Therefore, if a fault occurs, the estimate of $x$ is such that

$$
\| \hat{x}(t) - x(t) \| > \epsilon, \quad \forall t \geq t_\eta
$$

where $\epsilon$ is a positive constant.

Thus, the high-order sliding mode observer (11) enables to robustly estimate the observable part of the estimation error $\hat{x} - x$ while the hybrid observer (7) does not converge towards zero in the presence of fault $f_x$ due to Assumption 1. Indeed, in each time interval $[t_{k-\eta}, t_k)$, some observable parts of $e_x(t)$ [i.e. $\hat{x}(t)$] are influenced by the presence of a fault in case of occurrence. The estimation $\hat{x}(t)$ deviates from zero when a fault appears (see Theorem 3). Therefore, the norm of the estimated state observable parts $\| \hat{x}(t) \|$ is designed to be sensitive to a fault and acts as a residual signal for detection of faults. Based on this residual, a detector is used to alarm us from the appearance of the fault.

From Theorem 3, a fault on the continuous part is detected if $\| \hat{x}(t) \|$ exceeds a certain threshold value $T_h$. An index $F_{det}$ can be defined as follows

$$
F_{det} = \begin{cases} 
0 & \text{if } \| \hat{x}(t) \| \leq T_h \\
1 & \text{if } \| \hat{x}(t) \| > T_h
\end{cases}
$$

**Remark 1:** From the definition of the sliding mode observer (11), the vector $\hat{z}(t)$ is re-initialised at each switching time. Indeed, the size of this vector depends on the observability matrix $H_k$ and changes. The fault detector is used to detect an abnormal behaviour when $\| \hat{z}(t) \|$ exceeds a certain threshold value $T_h$ while removing the effect of this initialisation.
4 Simulation results on two practical examples

In this section, the fault detection algorithm based on sliding mode observer is applied to two different switched systems. Analytical studies are presented on the multicellular converter and on the boost converter (see Fig. 1) to detect faults on the continuous part of the system due to a variation of the capacitor values.

4.1 Application to the multicellular converter

4.1.1 Modelling: The multicellular converter is based on the combination of $p$ elementary cells of commutation. Its dynamics, with a load consisting in a resistance $R$ and an inductance $L$, can be expressed as

$$
\begin{align*}
\dot{V}_c &= \frac{1}{c_j(1+\Delta_j)} (S_j+1 - S_j), \quad j = 1, \ldots, p - 1 \\
I &= -\frac{R}{L} + \frac{E}{L} c_j - \sum_{j=1}^{p-1} \frac{V_c}{L} (S_j+1 - S_j)
\end{align*}
$$

(30)

where $I$ is the load current, $c_j$ is the capacitance, $\Delta_j$ is associated with a possible fault on the capacitor, $V_c$ is the voltage in the $j$th capacitor and $E$ is the voltage of the source. Only the output current $I$ is measured, $y = I$.

Hereafter, the study is realised on a three-cells converter ($p = 3$) with $E = 60 \, \text{V}$, $R = 200 \, \Omega$, $L = 1 \, \text{H}$, $c_1 = c_2 = 40 \, \mu\text{F}$, $c_3 = 0.01 \, \mu\text{F}$. Each commutation cell is controlled by a binary signal $S_i \in \{0, 1\}$, $i = 1, 2, 3$ using the pulse width modulation (PWM) strategy. Signal $S_i = 1$ means that the upper switch of the $i$th cell is ‘on’ and the lower switch is ‘off’ whereas $S_i = 0$ means that the upper switch is ‘off’ and the lower switch is ‘on’.

System (30) can be written as system (1) where $x = [V_{c_1} \, V_{c_2} \, I]^T$, $u = E$, $y = I$, $\sigma = 1 + \sum_{j=1}^3 2^{j-1} S_i \in \{1, \ldots, 8\}$ and $f_c \in \mathbb{R}^2$. Matrices of system (1) are defined as

$$
\begin{align*}
A_{\sigma(i)} &= \begin{bmatrix} 0 & 0 & (S_2 - S_1) \\ 0 & 0 & (S_3 - S_2) \\ -\frac{(S_2 - S_1)}{L} & -\frac{(S_3 - S_2)}{L} & \frac{c_1}{L} \end{bmatrix}, \\
B_{\sigma(i)} &= \begin{bmatrix} 0 & 0 & \frac{1}{L} \end{bmatrix}^T, \\
C_{\sigma(i)} &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, \\
K_{\sigma(i)} &= \begin{bmatrix} \frac{\Delta_1 (S_2 - S_1)}{c_1(1+\Delta_1)} & 0 & 0 \\ 0 & \frac{\Delta_2 (S_2 - S_1)}{c_2(1+\Delta_2)} & 0 \\ 0 & 0 & 0 \end{bmatrix}
\end{align*}
$$

Here, the objective is to detect faults on the continuous part of the system due to a variation of the capacitor values $c_1$ or $c_2$. Faults are given by the term $K_{\sigma(i)} f_c(t)$ with $\Delta_1 = \Delta_2 = 0.1$ and

$$
f_c(t) = \begin{cases} 
\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}^T & \text{if no fault occurs} \\
\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}^T & \text{if a fault on } c_1 \text{ occurs} \\
\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}^T & \text{if a fault on } c_2 \text{ occurs}
\end{cases}
$$

(31)

One can highlight that the observer matching condition is not satisfied since $\text{Rank}(K_{\sigma(i)}^T f_c(t)) \neq \text{Rank}(C_{\sigma(i)})$. Furthermore, computing the observability matrix for each subsystem, one can easily check that the subsystems associated with (30) are not observable. Nevertheless, three cycles can be calculated using the PWM control ($'1', '2', '3', '4', '5'$ or '$2', '3', '4', '5', '6', '7'$ or '$8', '4', '8', '6', '8', '7'$). This implies that the switching signal is persistent and system (30) is persistently observable according to Theorem 1 with $\eta = 3$.

4.1.2 Design of the observer-based fault detection scheme: To design the hybrid observer (7), using (9), matrices $Z_k$ become

$$
Z_1 = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}, \quad Z_2 = \begin{bmatrix} 0.01 & 1 \\ 0 & 1 \end{bmatrix}, \quad Z_3 = \begin{bmatrix} 0.02 & 0.7 \\ 0.01 & 0.02 \end{bmatrix}, \quad Z_4 = \begin{bmatrix} 0 & 0 \\ 0.01 & 1 \end{bmatrix}, \quad Z_5 = \begin{bmatrix} 0.01 & 0 \\ 0.01 & 0 \end{bmatrix}, \quad Z_6 = \begin{bmatrix} -0.02 & 0.7 \\ -0.02 & 0.7 \end{bmatrix}, \quad Z_7 = \begin{bmatrix} -0.01 & -1 \\ 0 & 0 \end{bmatrix}, \quad Z_8 = \begin{bmatrix} -0.01 & -1 \\ 0 & 0 \end{bmatrix}
$$

Similarly, one can also determined the matrices $W_k$ associated with the orthonormal basis of $\text{ker}(H_{\eta})$. Notice that modes ‘1’ and ‘8’ have a single element observable and the other modes have two elements observable.

The estimation of the state observable $\hat{z}(t)$, which acts as the residual signal, is given by observer (11) with matrices

$$
S_{1,8} = -200, \quad S_{2,4,7} = \begin{bmatrix} 133 \\ -25000 \end{bmatrix}, \quad S_3 = \begin{bmatrix} 133 \\ 3 \end{bmatrix}, \quad S_5 = \begin{bmatrix} 133 \\ -25000 \end{bmatrix}, \quad S_6 = \begin{bmatrix} -1200 \\ -35000 \end{bmatrix}
$$

and

$$
R_1 = -1, \quad R_2 = -0.03, \quad R_3 = -1, \quad R_4 = -1, \quad R_5 = -1, \quad R_6 = -0.03
$$

Fig. 1 Practical examples to test our fault detection algorithm

a Multicellular converter

b Boost converter
A fault on \( c \) shows that the estimation error converges towards zero when no fault occurs (before \( t = 0.25 \text{ s} \)) in finite time. However, when a fault appears, the error does not converge towards zero. The norm of the estimated state observable parts \( \| \hat{z}(t) \| \) is influenced by the presence of fault in case of occurrence.

The high-order sliding mode observer (11) enables to robustly estimate the observable part of the estimation error \( x - \hat{x} \). The norm of the estimated state observable parts \( \| \hat{z}(t) \| \), depicted in Fig. 3a, is sensitive to faults and acts as a residual signal for detection of faults. Indeed, it well deviates from zero when a fault appears.

4.1.3 Simulation results: The initial state of the converter is \( x(t_0) = [0, 0, 0]^T \) and the minimal dwell time is \( T_b = 2 \text{ ms} \). The initial state of the observer is arbitrary chosen as \( \hat{x}(t_0) = [20, 10, 0.17] \). In the following, two scenarios are considered: a fault occurs on \( c_1 \) (resp. on \( c_2 \)):

- A fault on \( c_1 \) occurs from 0.25 s. Fig. 2 shows the evolution of the continuous state \( [V_{c_1}, V_{c_2}, I] \), its estimation and the estimation error \( \hat{x} - x \). One can notice that the estimation error converges towards zero when no fault occurs (before \( t = 0.25 \text{ s} \)) in finite time. However, when a fault appears, the error does not converge towards zero. Hence, some observable parts of \( c_1(t) \) [i.e. \( z(t) \)] are influenced by the presence of fault in case of occurrence.

4.2 Application to the boost converter

4.2.1 Modelling: The boost circuit, shown in Fig. 1b, consists of a capacitor \( C \), an inductance \( L \) and its internal resistor \( R_L \), a voltage source \( E \), a load resistance \( R \) and ideal switches \( S \). Its dynamics depends on the discrete state of the switch \( S \in [0, 1] \). \( S = 0 \) corresponds to the nominal configuration of the converter whereas \( S = 1 \) is the reversed state of the cell. It is assumed that the current in the inductance \( I_L \) is not measurable but a voltage sensor can provide the voltage \( V \) in the capacitor. Hence, it can be modelled by two modes of operation following system (1) with \( x = [i_L, V_c]^T \), \( u = E, y = V_c, \sigma = 1 + S \in [1, 2], f_c \in \mathbb{R}^2 \) and the effect of this initialisation. It has been implemented based on the detection index \( F_{det} \) (see Fig. 3b). The transient period (i.e. \( t \leq 0.03 \text{ s} \)) which corresponds to the time needed for the convergence of the hybrid observer is removed from the detection analysis. One can see that the fault acting on \( c_1 \) is well detected.

- A fault on \( c_2 \) occurs from 0.25 s. Fig. 4a shows that the estimation error converges towards zero when no fault occurs (before \( t = 0.25 \text{ s} \)) in finite time. However, when a fault appears, the error does not converge towards zero. The norm of the estimated state observable parts \( \| \hat{z}(t) \| \), depicted in Fig. 4b, is sensitive to faults and acts as a residual signal for detection of faults. Indeed, it well deviates from zero when the fault appears.
Fig. 4 Fault acting on c2 for the three cells converter
a Zoom on the state estimation error \( \hat{x}(t) - x(t) \)
b Residual signal \( |\hat{z}(t)| \)

Fig. 5 Estimation errors on the continuous state in the presence of fault on C for the boost converter
a State \( x(t) = [i_c, v_c]^T \) and its estimate
b Zoom on the state estimation

Fig. 6 Fault acting on C for the boost converter
a Zoom on the state estimation error \( \hat{x}(t) - x(t) \)
b Residual signal \( |\hat{z}(t)| \) in the presence of fault on C

matrices [21]

\[
A_1 = \begin{bmatrix}
\frac{-R_L}{L} & 0 \\
0 & \frac{1}{RC}
\end{bmatrix}, \quad A_2 = \begin{bmatrix}
\frac{R_L}{L} & -\frac{1}{L} \\
\frac{1}{C} & -\frac{1}{RC}
\end{bmatrix}
\]

\[
B_{1,2} = \begin{bmatrix}
\frac{1}{L} \\
0
\end{bmatrix}^T, \quad C_{1,2} = \begin{bmatrix}
0 & 1
\end{bmatrix}
\]

\[
K_1 = \begin{bmatrix}
0 & 0 \\
0 & \frac{\Delta_v}{RC(1 + \Delta_v)}
\end{bmatrix}
\]
\[
K_2 = \begin{bmatrix}
0 & 0 \\
\Delta c & \Delta c
\end{bmatrix}
\]

\(D_{1,2} = 0\)

where \(\Delta c\) is associated with a possible fault on the capacitor.

The parameters of the system are set as \(E = 25\) V, \(R = 20\) \(\Omega\), \(L = 20\) mH, \(R_L = 0.03\) \(\Omega\), \(C = 100\) \(\mu\)F. Here, the objective is to detect a fault on the capacitor value. Fault is given by the term \(K_{\eta(t)}f_c(t)\) with \(\Delta c = -0.1\) and

\[
f_c(t) = \begin{bmatrix}
0 \\
x(t)
\end{bmatrix}^T \text{ if no fault occurs}
\begin{bmatrix}
0 \\
x(t)
\end{bmatrix} \text{ if a fault on C occurs}
\]

One should notify that condition (3) does not hold since \(r_C > p\) and \(\text{Rank}(K_{\eta(t)}) \neq r_C\). Furthermore, computing the observability matrix for each subsystem, one can easily check that mode 1 is not observable. Nevertheless, the commutations operated by the PWM strategy provide a persistent switching signal. System, defined by matrices (33), is persistently observable according to Theorem 1 where \(\eta = 2\).

4.2.2 Simulation results: The initial state of the converter (resp. observer) is \(x(t_0) = [0, 0]^T\) (resp. \(\hat{x}(t_0) = [1, 10]^T\)). In this simulation, a fault on C occurs from 0.1 s. Fig. 5 shows the evolution of the continuous state \([L_c, V_c]^T\) and its estimation. It can be highlighted that the estimation error \(\hat{x} - x\) converges towards zero when no fault occurs (before \(t = 0.1\) s) in finite time. When a fault appears, the residual signal \(\|\tilde{z}\|\), depicted in Fig. 6, shows an abnormal behaviour of the system.

5 Conclusion

A fault detection scheme based on higher-order sliding mode observer has been proposed for a class of switched linear systems when the observer matching condition is not satisfied. Without using decoupling transformations, a residual signal, based on reduced order sliding mode observers, has been introduced for the detection of faults which occur on the continuous part of the system. Studies on the cascade multilevel converter and on the boost converter have been carried out to detect fault corresponding to a capacitor ageing.

6 References