4.0 Torque Sensors and Tactile Sensing

Sensing of *torque and force* is useful in many applications, they include:

- In the control of fine motions and in assembly tasks, where a small error can cause large damaging forces or performance degradation.
- In control systems that are not fast enough when motion feedback alone is employed, where force feedback and feedforward control can be used to improve accuracy and bandwidth.

**Common methods of torque sensing** include:

- Measuring strain in a sensing member between the drive element and the driven load, using a strain-gage bridge.
- In electric motors, measuring the field current or armature current, which produces motor torque.
- Measuring torque directly, using piezoelectric sensors, for example.
- Measuring the angular acceleration caused by the unknown torque in a known inertia element.

4.9 Torque Sensors

### 4.9.1 Strain-Gage Torque Sensors

The most straightforward method of torque sensing is to connect a torsion member between the drive unit and load in series and to measure torque in it.

For a circular shaft, the torque-strain relationship is:

\[ \varepsilon = \frac{r}{2GJ} T \]

where:
- \( \varepsilon \) is principal strain (45° to axis) at a radius \( r \)
- \( J \) is polar moment of area of cross-section of the member
- \( G \) is shear modulus of the material
- \( r \) is the radius of the shaft
- \( T \) is torque transmitted through the member

Also, the shear stress \( \tau \) at a radius \( r \) of the shaft is given by:

\[ \tau = \frac{T}{J} \]

Using the general bridge equation:

\[ \frac{\delta V_o}{V_{ref}} = k \frac{5}{4} \frac{S_s}{S} \varepsilon \]

Here, \( \varepsilon = \frac{r}{2GJ} T \Rightarrow \frac{\delta V_o}{V_{ref}} = \frac{kTTS_s}{8GJ} \Rightarrow T = \frac{8GJ}{kTS_s} V_{ref} \)

Strain gages are assumed to be mounted along a principal direction. *Three possible configurations* are shown below:

![Strain gage configurations for a circular shaft torque sensor](image)
In configurations (a) and (b), only two strain gages are used, and the bridge constant $k = 2$.

Both axial and bending loads are compensated with given configurations -- resistance in both gages will be changed by the same amount -- which cancels out up to first-order for the bridge circuit connections.

Configuration (c) has two pairs of gages, mounted on the two opposite surfaces of the shaft -- the bridge constant is doubled in this configuration -- sensor self-compensates for axial and bending loads up to first order.

### 4.9.2 Design Considerations

- Two conflicting requirements in the design of torsion element for torque sensing are **sensitivity and bandwidth**
  - Element has to be flexible to get an acceptable sensor sensitivity
  - Increase in flexibility results reduction of the overall stiffness

Overall stiffness $K_{old}$ before connecting to the torsion element is,

$$\frac{1}{K_{old}} = \frac{1}{K_{m}} + \frac{1}{K_{L}}$$

After connecting the torsion member is,

$$\frac{1}{K_{new}} = \frac{1}{K_{m}} + \frac{1}{K_{L}} + \frac{1}{K_{s}}$$

$K_{m}$ is equivalent stiffness of the drive unit (motor)

$K_{L}$ is equivalent stiffness of the load

$K_{s}$ is stiffness of the torque-sensing element

### 4.9.2.1 Strain Capacity of the Gage

The maximum strain handled by a strain-gage element is limited by factors such as strength, creep problems and hysteresis.

Strain capacity limit specified by strain-gage manufacturer is not exceeded

$$\frac{r}{2GJ} T_{max} < \varepsilon_{max} \Rightarrow J > \frac{r}{2G} \frac{T_{max}}{\varepsilon_{max}}$$

where, $\varepsilon_{max}$ and $T_{max}$ are specified

### 4.9.2.2 Strain-Gage Nonlinearity Limit

For large strains, the characteristic equation of a strain-gage becomes increasingly non-linear.

For a specified nonlinearity, an upper limit for strain can be determined,

$$\frac{r}{2GJ} T_{max} = \varepsilon_{max} \leq \frac{N_{p} S_{1}}{50S_{2}} \Rightarrow J \geq \frac{25rS_{2}}{G5S_{1}} \frac{T_{max}}{N_{p}}$$

where, $N_{p}$ and $T_{max}$ are specified

### 4.9.2.3 Sensitivity Requirement

- Strain gage bridge is provided to a differential amplifier, takes the difference and amplifies the output
- Large gain increases the susceptibility of the amplifier to saturation.

Sensor sensitivity is acceptable in terms of the output signal level of the differential amplifier in the bridge circuit

$$v = K_{a} \delta v_{o} \quad ; \quad v_{o} \leq \frac{K_{a} k S_{s} r v_{ref} T_{max}}{8G} \Rightarrow J \leq \frac{K_{a} k S_{s} r v_{ref} T_{max}}{8G} v_{o}$$

where $K_{a}$ is amplifier gain, $v_{o}$ and $T_{max}$ are specified
4.9.2.4 Stiffness Requirement

The overall stiffness of the system is constrained by speed of response (bandwidth) and steady-state error.

For a shaft of length $L$ and radius $r$, a twist angle of $\theta$ corresponds to shear strain of, $\gamma = \frac{r\theta}{L}$, and shear stress is $\tau = \frac{G\theta}{L}$; stiffness is $K_s = \frac{r\theta}{G}$.

\[ J \geq \frac{L}{G} K \]

where $K$ is specified

**Design criteria for a strain-gage element**

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Specification</th>
<th>Governing formula for polar moment of area ($J$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strain capacity of strain-gage element</td>
<td>$\varepsilon_{\text{max}}$ and $T_{\text{max}}$</td>
<td>$J &gt; \frac{r T_{\text{max}}}{2G \varepsilon_{\text{max}}}$</td>
</tr>
<tr>
<td>Strain-gage nonlinearity</td>
<td>$N_p$ and $T_{\text{max}}$</td>
<td>$J &gt; \frac{25rS_2 T_{\text{max}}}{G S_1 N_p}$</td>
</tr>
<tr>
<td>Sensor sensitivity</td>
<td>$v_o$ and $T_{\text{max}}$</td>
<td>$J \leq \frac{K_s k S r^2 v_{\text{ref}} T_{\text{max}}}{8G v_o}$</td>
</tr>
<tr>
<td>Sensor stiffness (bandwidth and gain)</td>
<td>$K$</td>
<td>$J \geq \frac{L}{G} K$</td>
</tr>
</tbody>
</table>

4.9.3.1 Direct-Deflection Torque Sensor

- Direct measurement of the twist angle can be used to measure torque
- Proximity probes produce pulse sequences as the shaft rotates
- The phase shift of the two signals determines the angular deflection which is a measure of the transmitted torque
- Both the magnitude and the direction of the torque can be measured

4.9.3.2 Variable Reluctance Torque Sensor

- This sensor operates like a differential transformer
- Torque sensing element is a ferromagnetic tube with two slits placed in the direction of principle stresses
- When a torque is applied, one slit opens and other closes causing a change in reluctance
- Output voltage is a measure of the transmitted torque

**Example (1):**

Consider a rigid load, which has a polar moment of inertia $J_L$, and driven by a motor with a rigid rotor, which has inertia $J_m$. A torsion member of stiffness $K_s$ is connected between the rotor and the load, as shown in Figure, to measure the torque transmitted to the load. Determine the transfer function between the motor torque $T_m$ and the twist angle $\theta$ of the torsion member. What is the torsional natural frequency $\omega_n$ of the system? Discuss why the system bandwidth depends on $\omega_n$. Show that bandwidth can be improved by increasing $K_s$, by decreasing $J_m$ or $J_L$.

**System model**

Free-body diagram
Solution

From the free-body diagram, the equations of motion can be written as,

Motor: \( T_m = J_m \ddot{\theta}_m + K_s (\theta_m - \theta_L) \);  
Load: \( K_s (\theta_m - \theta_L) = J_L \ddot{\theta}_L \)

From the above, \( \ddot{\theta}_m - \ddot{\theta}_L = \frac{T_m}{J_m} - \frac{K_s}{J_m} (\theta_m - \theta_L) - \frac{K_s}{J_L} (\theta_m - \theta_L) \)

Let, \( \theta = \theta_m - \theta_L \Rightarrow \ddot{\theta} + K_s \left( \frac{1}{J_m} + \frac{1}{J_L} \right) \theta = \frac{T_m}{J_m} \)

Hence, \( G(s) = \frac{\theta(s)}{T_m(s)} = \frac{1/J_m}{s^2 + K_s \left( \frac{1}{J_m} + \frac{1}{J_L} \right)} \Rightarrow \omega_n = \sqrt{\frac{K_s}{\frac{1}{J_m} + \frac{1}{J_L}}} \)

- System bandwidth improves when \( \omega_n \) is increased. Hence, \( \omega_n \) is a measure of system bandwidth
- It can be observed that \( \omega_n \) (and the bandwidth) increases when \( K_s \) is increased and \( J_m \) or \( J_L \) is decreased
- If a gearbox is added to the system, the equivalent inertia increases and the stiffness decreases, resulting reduction in bandwidth

Example (2):

A joint of a direct-drive robotic arm is sketched in below Figure. Note that the rotor of the drive motor is an integral part of the driven link, without the use of gears or any other speed reducers. Also, the motor stator is an integral part of the drive link. A tachometer measures the joint speed (relative), and a resolver measures the joint rotation (relative). Gearing is used to improve the performance of the resolver. Neglecting mechanical loading from sensors and gearing, but including bearing friction, sketch the torque distribution along the joint axis. Suggest a location (or locations) for measuring the net torque transmitted to the driven link using a strain gage torque sensor

Example (3):

Consider the design of a tubular torsional element. The following design specifications are given: \( \varepsilon_{\text{max}} = 3000 \mu \varepsilon \); \( N_p = 5\% \); \( \nu_0 = 10 V \); and for a system bandwidth of 50Hz, \( K = 2.5 \times 10^3 \) N.m/rad. A bridge with four active strain gages is used to measure torque in the torsion element. The following parameter values are provided:

1. For strain gages: \( S_s = S_1 = 115, S_2 = 3500 \)
2. For the torsion element: \( r = 2 \) cm, \( G = 3 \times 10^{10} \) N/m², \( L = 2 \) cm
3. For the bridge circuitry: \( \nu_{\text{ref}} = 20 V, K_a = 100 \)

The maximum torque that is expected is \( T_{\text{max}} = 10 \) Nm. Using these values, design a torsion element for the sensor.

Solution

We can compute the \( J \) using each of the four criteria given in table earlier.

1. For \( \varepsilon_{\text{max}} = 3000 \mu \varepsilon \); \( J = \frac{r}{2G} \frac{T_{\text{max}}}{\varepsilon_{\text{max}}} = \frac{0.02 \times 10^{-3}}{2 \times 3 \times 10^{10} \times 3 \times 10^{-3}} m^4 = 1.11 \times 10^{-9} m^4 \)
Solution (Contd..):

2. For \( N_p = 5; J = \frac{25S_2 T_{\text{max}}}{GS_1 N_p} = \frac{25 \times 0.02 \times 3500 \times 10}{3 \times 10^{10} \times 115 \times 5} \ m^4 = 1.01 \times 10^{-9} \ m^4 \)

3. For \( v_0 = 10V; J = \frac{K a k S_2 v_{\text{pref}} T_{\text{max}}}{8G v_0} = \frac{100 \times 4 \times 115 \times 0.02 \times 20 \times 10}{8 \times 3 \times 10^{10} \times 10} \ m^4 = 7.67 \times 10^{-8} \ m^4 \)

4. For \( K = 2.5 \times 10^3 \ N.m/\text{rad}; J = \frac{L}{K} = \frac{0.02 \times 2.5 \times 10^3}{3 \times 10^{10}} \ m^4 = 1.67 \times 10^{-9} \ m^4 \)

It follows that for an acceptable sensor, we should satisfy,

\[ J \geq (1.11 \times 10^{-9}) \text{ and } (1.01 \times 10^{-9}) \text{ and } (1.67 \times 10^{-9}) \text{ and } (7.67 \times 10^{-8}) \ m^4 \]

We pick \( J = 7.67 \times 10^{-8} \ m^4 \) so that the tube thickness is sufficiently large to transmit load without buckling or yielding.

4.10 Tactile Sensing

- **Tactile sensing** is usually interpreted as **touch sensing**
  -- a force distribution is measured using a closely spaced array of force sensors and exploiting the **skin-like properties** of the sensor array

- Tactile sensing is important in two types of operations
  - **Grasping:** object has to be held in stable manner without being damaged
  - **Object identification:** recognizing or determining the shape, location, and orientation of an object as well as detecting surface properties

4.10.1 Typical Sensor Requirements

- Spatial resolution of about 2 mm
- Force resolution (sensitivity) of about 2 gm
- Force capacity (maximum touch force) of about 2 kg
- Response time of 5 ms or less
- Durability under harsh working conditions etc.,

4.10.2 Construction and Operation of Tactile Sensors

- The **touch surface** of a tactile sensor is usually made of an **elastomeric pad or flexible membrane**

- Principle of tactile sensor differs depending on whether **distributed force is sensed** or **deflection of tactile surface** is measured. Methods include,
  - Use a closely spaced set of strain gages to sense the distributed force
  - Use a conductive elastomer as the tactile force. The change in its resistance determines the distributed force
  - Use a closely spaced array of deflection sensors to determine the deflection profile of the tactile surface

- Since force and deflection are related thru a constitutive law for tactile sensor (touch pad), only one type of measurement is needed in sensing

- The skin-like membrane itself can be made from a conductive elastomer, whose changes in resistance can be sensed

4.10.3 Optical Tactile Sensors

- More light is received by the light receiver when the reflecting surface is at **Position 2** than when it is at **Position 1**
- If the reflecting surface touches the light source, light becomes completely **blocked off**, and no light reaches the receiver
- Can determine the position \( (x) \) using proximity-intensity curve
4.10.5 Dexterity
Motion dexterity = \( \frac{\text{number of degrees of freedom in the device}}{\text{motion resolution of the device}} \)
Force dexterity = \( \frac{\text{number of degrees of freedom in the device}}{\text{force resolution of the device}} \)

4.10.6 A Strain-Gage Tactile Sensor

![Schematic of a strain-gage point-contact sensor](image)

**Example (4):**
A tactile sensor pad consists of a matrix of conductive elastomer elements. The resistance \( R_t \) in each tactile element is given by, \( R_t = a/F_t \), where \( F_t \) is the tactile force applied to the element and \( a \) is a constant. The circuit shown below is used to acquire the tactile sensor signal \( v_o \) which measures the local tactile force \( F_t \). The entire matrix of tactile elements may be scanned by addressing the corresponding elements through an appropriate switching arrangement. For the signal acquisition circuit, obtain a relationship for the output voltage \( v_o \) in terms of the parameters \( a, R_t \) and others if necessary, and the variable \( F_t \). Show that \( v_o = 0 \) when the tactile element is not addressed (circuit is switched to ref. voltage 2.5 V).

**Solution**
Define, \( v_i = \text{input to the circuit (2.5 V or 0.0 V)} \)
\( v_{o1} = \text{output of the first op-amp} \)

Properties of an op-amp: 1) voltages at the two input leads are equal, 2) currents through the two input leads are zero.

Current balance at A: \( \frac{5.0 - v_i}{R} = \frac{v_{i} - v_{o1}}{R} \Rightarrow v_{o1} = 2v_i - 5.0 \) (i)

Current balance at B: \( \frac{v_{o1} - 0}{R_t} = \frac{0 - v_o}{R_o} \Rightarrow v_o = -v_{o1} \frac{R_o}{R_t} \) (ii)

Substitute eq. (i) into eq. (ii) and also the expression for \( R_t \), we get
\( v_o = -v_{o1} \frac{R_o}{a} F_t (5.0 - 2v_i) \)

Substitute two switching values for \( v_i \). We have
\( v_o = \frac{5.0R_o}{a} F_t \) when addressed
\( = 0 \) when reference

- It can be used to determine the size and location of a point-contact force
- A square plate of length \( a \) is simply supported by frictionless hinges
- A point force \( P \) applied to the plate can be determined

The location of force \( P \) is given by the coordinates \( (x, y) \) in the Cartesian coordinate system \( (x, y, z) \). The load cell reading at location \( i \) is \( R_i \).

Equilibrium in \( z \)-direction gives force balance, \( P = R_1 + R_2 + R_3 + R_4 \)
Equilibrium about \( y \)-axis gives moment balance,
\( Px = R_2a + R_3a \Rightarrow x = \frac{a}{P} (R_2 + R_3) \)

Similarly, equilibrium about \( x \)-axis gives,
\( y = \frac{a}{P} (R_3 + R_4) \)

From the above, the force \( P \) (direction as well as magnitude) and its location \((x, y)\) are determined by load cell readings

Typical values for plate length \( a \) and force \( P \) are 5 cm and 10 kg
Example (5):
In a particular parts-mating process using the principle of strain-gage tactile sensor described earlier, suppose that the tolerance on the measurement error of the force location in limited to $\delta r$. Determine the tolerance $\delta F$ on the load-cell error.

Solution

Equations for parts-mating process provided earlier are,

\[ P = R_1 + R_2 + R_3 + R_4 \]
\[ Px = R_2 a + R_3 a \]

Hence, $\delta P = \delta R_1 + \delta R_2 + \delta R_3 + \delta R_4$ & $P \delta x + x \delta P = a \delta R_2 + a \delta R_3$

Then, it becomes,

\[ \delta x = \frac{a}{P} (\delta R_2 + \delta R_3) - \frac{x}{P} (\delta R_1 + \delta R_2 + \delta R_3 + \delta R_4) \]

Note that $x$ lies between 0 and $a$, and each $\delta R_i$ can vary up to $\pm \delta F$. Hence, the largest error in $x$ is given by \( \frac{2a}{P} \delta F \). This is limited to $\delta r$. Hence,

\[ \delta r = \frac{2a}{P} \delta F \text{ (or) } \delta F = \frac{P}{2a} \delta r \]

which gives tolerance on force error.

Summary:

- In most applications, sensing is done by detecting an effect of torque or the cause of torque.
- The most straightforward method of torque sensing is to connect a torsion member between the drive unit and load in series.

Design considerations

1) Strain capacity of the gage, \( J > \frac{r T_{max}}{2G} \)
2) Strain-gage nonlinearity limit, \( J \geq \frac{25r S_2 T_{max}}{G S_1} \)
3) Sensitivity requirement, \( J \leq \frac{K_a k S_r T_{ref} T_{max}}{8G} \)
4) Stiffness requirement, \( J \geq \frac{L}{G} \)

Summary (Contd.):

- Tactile sensing is usually interpreted as touch sensing, in which, a force distribution is measured.
- Tactile sensor requirements
  -- Spatial resolution of about 2 mm
  -- Force resolution (sensitivity) of about 2 gm
  -- Response time of 5 ms or less
  -- Durability under harsh working conditions etc.,
- Strain-gage tactile sensor
  Equilibrium in the $z$- direction, force balance, $P = R_1 + R_2 + R_3 + R_4$
  Equilibrium about $x$- axis, $x = \frac{a}{P} (R_2 + R_3)$
  Equilibrium about $y$- axis, $y = \frac{a}{P} (R_3 + R_4)$